WISE 2026/27 RESEARCH SEMINAR: EFIMOV K-THEORY

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The goal of this seminar is to learn Efimov's continuous K-theory of dualizable stable categories, following [Efi25]. We will first study compactly assembled and dualizable categories, and construct the extension of localizing invariants to dualizable categories. We will then go through the computation of the continuous K-theory of continuous posets and of locally compact Hausdorff spaces.

(1) **Introduction and distribution of talks (13.10)**
(2) **Compactly assembled and dualizable categories (20.10)** [Efi25, §1.4] Give the definitions of compactly assembled categories and of dualizable (stable presentable) categories, and explain the relationship between the two ([Efi25, Proposition 1.11], see also [Lur18, §21.1, §D.7]). Define the categories CompAss and $\mathrm{Cat}_{\mathrm{st}}^{\mathrm{dual}}$. Prove that dualizable categories arise as kernels of localizations of compactly generated categories [Efi25, Proposition 1.17]. Prove that compactly assembled categories are $\aleph_1$-compactly generated [Efi25, Proposition 1.24].
(3) **Examples (27.10)** [Efi25, §1.5–1.6] Disscuss Examples 1.27 (almost modules), 1.28 and 1.32 (sheaves on LCH spaces), 1.29 (Clausen–Scholze nuclear modules), 1.31 (extended real line), and 1.33 (seminormed vector spaces) in [Efi25] in more or less details, using the provided references. For the case of sheaves of spectra on a LCH space, explain why they form a dualizable category by taking the sheaf/K-sheaf corespondence for granted (this will be discussed in detail in Talk 12). Describe the left adjoint to the inclusion $\mathrm{Cat}_{\mathrm{st}}^{\mathrm{dual}} \hookrightarrow \mathrm{CompAss}$.
(4) **Compact morphisms and AB6 (03.11)** [Efi25, §1.8–1.9] Define compact morphisms and prove the corresponding characterization of dualizable categories and strongly continuous functors [Efi25, Theorem 1.45, Proposition 1.46]. Explain the relation between compact morphisms and wavy arrows [Efi25, Proposition 1.48]. Explain the AB6 axiom and show that it is equivalent to dualizability [Efi25, Proposition 1.53].
(5) **Calkin categories and colimits of dualizable categories (10.11)** [Efi25, §1.10–1.12]
(6) **Limits of dualizable categories (17.11)** [Efi25, §1.14–1.15]
(7) **Semiorthogonal decompositions and infinite products (24.11)** [Efi25, §1.13, §1.17]
(8) **Flat presentable categories (01.12)** [Efi25, §2] Show that a stable presentable category is dualizable if and only if it is flat.
(9) **Localizing invariants of dualizable categories (08.12)** [Efi25, §4.1–4.3]
(10) **Sheaves on the real line and applications (15.12)** [Efi25, §4.4–4.5]
(11) **Continuous posets (22.12)** [Efi25, §5]
(12) **Sheaves on locally compact Hausdorff spaces (12.01)** [Efi25, §6.2] See also [Lur17, §7.3.4].
(13) **K-theory of locally compact Hausdorff spaces (19.01)** [Efi25, §6.1, §6.3]
(14) **Urysohn's lemma (26.01)** [Efi25, Appendices C and D] Show that $\mathrm{Cat}_{\mathrm{st}}^{\mathrm{dual}}$ is $\aleph_1$-compactly generated by the single object $\mathrm{Shv}_{\geq 0}(\mathbb{R}, \mathrm{Sp})$.
(15) **TBD (02.02)**

References

[Efi25] A. I. Efimov, *K-theory and localizing invariants of large categories*, 2025, arXiv:2405.12169v3
[Lur17] J. Lurie, *Higher Topos Theory*, April 2017, http://www.math.harvard.edu/~lurie/papers/HTT.pdf
[Lur18] ———, *Spectral Algebraic Geometry*, February 2018, http://www.math.harvard.edu/~lurie/papers/SAG-rootfile.pdf